Degenerated hexahedral Whitney elements for electromagnetic fields computation in multi-layer anisotropic thin regions

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In this article, the use of degenerated hexahedral Whitney elements method in the modeling of multi-layer anisotropic regions is presented. These elements degenerated from hexahedral elements are used to model laminated composites in the scale of one ply taking into account the circulation of eddy-currents along the thickness of the material. Eddy-current problem is solved using $\boldsymbol{A}-\phi$ formulation.

***Index Terms*—Eddy currents, Degenerated Whitney element, Thin Region, Anisotropic material, Laminated CFRP.**

I. Introduction

A laminated Carbon Fiber Reinforced Polymer (CFRP) composite is a stacking of several anisotropic unidirectional plies (Fig. 1). In each ply, the carbon fibers are oriented in a same direction. The orientation of each ply in a laminate may be different. The stacking sequence describes the orientation of all plies of the laminate. The thickness of a ply is a few tenths of mm, the size of an entire plate is generally in order of few dozen plies thickness by some meters long.

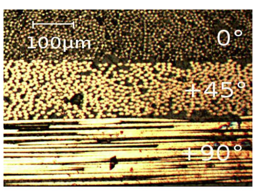


Fig. 1. Micrographic section of a composite.

The distribution of eddy-currents in these materials depends on the stacking sequence. In addition, the anisotropy of the material allows the circulation of eddy-currents along the thickness of the laminate. Modeling requires then a model in the scale of one ply of the composite in which each ply is considered by its own three-dimensional electrical conductivity tensor which depends on its orientation [1]. The laminate region is very thin and highly anisotropic, if the laminate is meshed, it may lead to some numerical issues due to the distortion of the mesh and the anisotropy of constitutive law. In this case, orthogonal hexahedral elements may be useful [2]. However, for complex systems, the computation time of this model may be high.

To avoid mesh in thin regions, some methods replaces the thin region by its median surface mesh. In order to take into account the variation of the field across its thickness, these methods use assumptions *a priori* determined. For example, in [3], the electrical conductivity along the thickness of the material ($\sigma_z = 0$) is neglected. In [4] [5], the behavior of the fields in the thin region is predetermined and imposed on the surfaces of the region through a boundary condition. The later neglects the circulation of eddy-currents along the thickness of the material. In [6], Ren proposes a family of degenerated prismatic Whitney elements that can be used to discretize the fields in single layer thin regions. This approach allows modeling the circulation of induced currents along the thickness of the material.

In the following, an extension of this approach to anisotropic multi-layer regions is presented. Double layers shell elements degenerated from hexahedron elements will be firstly shown. Then, this is followed by the use of these elements in the multi-layer laminated composite domains. Simulation results will be compared with classical method.

II. Nodal and Edge Shell Elements

The linear shape functions defined on a quadrilateral element (in 2D):

$$\lambda_1 = (1-u)(1-v)/4, \lambda_2 = (1+u)(1-v)/4 \quad (1)$$

$$\lambda_3 = (1+u)(1+v)/4, \lambda_4 = (1-u)(1+v)/4 \quad (2)$$

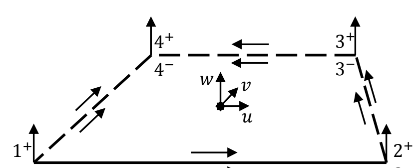


Fig. 2. Degenerated shell element of a regular hexahedron.

If we denote W_n^{2D} the nodal elements defined on the quadrilateral mesh of the upper and lower surfaces of the thin region, the degenerated nodal elements can be determined by:

$$W_{nS}^{-,+} = (W_n^{2D}\beta^-, W_n^{2D}\beta^+) \quad (3)$$

where $\beta^- = (1-w)/2$ and $\beta^+ = (1+w)/2$ are the interpolation functions along the thickness of the region (Fig. 2).

The degenerated edge elements are written as follows:

$$\boldsymbol{W}_{aS}^{-,+,\pm} = (\boldsymbol{W}_a^{2D}\beta^-, \boldsymbol{W}_a^{2D}\beta^+, W_n^{2D}\boldsymbol{n}/\varepsilon) \quad (4)$$

$\boldsymbol{W}_a^{2D}$ is the set of edge elements defined on a quadrilateral whose shape functions can be determined as follows:

$$\boldsymbol{W}_{ij}^{2D} = \lambda_i \boldsymbol{\nabla}(\lambda_i + \lambda_l) - \lambda_j \boldsymbol{\nabla}(\lambda_j + \lambda_k) \tag{5}$$

In these formulas, the node l is directly associated with the node i by an common edge and not related to node j, the node k is linked with the node j by a common edge and not related to the node i.

III. $\boldsymbol{A} - \phi$ FORMULATION IN ANISOTROPIC MULTI-LAYER REGION

In the case of a multi-layer region, it is necessary to create double-layer quadrilateral meshes that represent each layer (ply) of the region. The integrals calculated on the volume of each layer are transformed into surface integrals calculated on both surfaces of the corresponding double-layer quadrilateral mesh. The coefficients associated with the common edges and common nodes between two double-layer meshes is the contribution of those calculated in each mesh. The left-hand side of the $\boldsymbol{A} - \phi$ formulation [7] can be determined by:

$$\begin{aligned} &\textstyle\sum_{i=1,k} \int_{\Omega_c^i} \frac{1}{[\mu]_i} (\boldsymbol{\nabla} \times \boldsymbol{W}_a)(\boldsymbol{\nabla} \times \boldsymbol{A}) d\Omega = \\ &\textstyle\sum_{i=1,k} \int_\Gamma \frac{1}{\varepsilon_i \mu_{z,i}} [\boldsymbol{n} \times \boldsymbol{W}_a^{2D}][\boldsymbol{n} \times \boldsymbol{A}] d\Gamma + \\ &\textstyle\sum_{i=1,k} \int_\Gamma \varepsilon_i \frac{1}{[\mu]_{xy,i}} \left(\int_{-1}^{+1} < \boldsymbol{\nabla} \times \boldsymbol{W}_a^{2D} > \right. \\ &\qquad \left. < \boldsymbol{\nabla} \times \boldsymbol{A} > dw \right) d\Gamma \end{aligned}$$

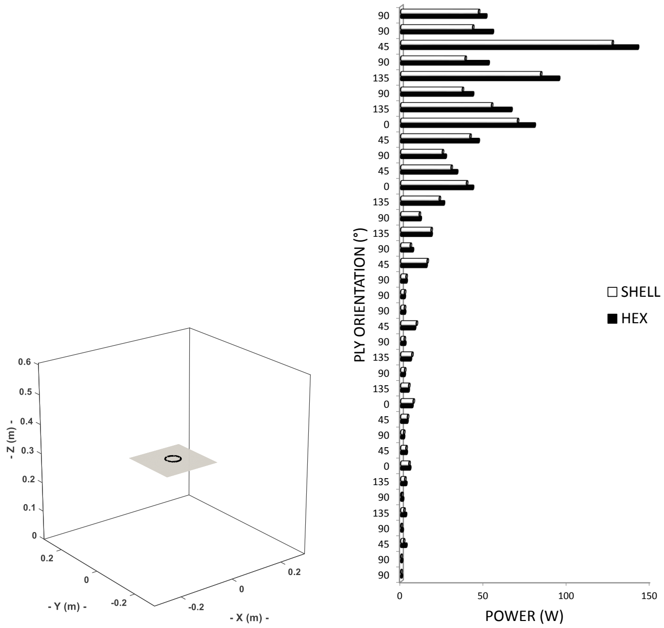
$$\begin{aligned} &\textstyle\sum_{i=1,k} \int_{\Omega_c^i} [\sigma]_i \left(\boldsymbol{W}_a + \boldsymbol{\nabla} W_n\right) \left(\boldsymbol{A} + \boldsymbol{\nabla}\phi\right) d\Omega = \\ &\textstyle\sum_{i=1,k} \int_\Gamma \frac{\sigma_{z,i}}{\varepsilon_i} \left(\varepsilon_i W_{an} + [W_n^{2D}]\right) \left(\varepsilon_i A_n + [\phi]\right) d\Gamma + \\ &\textstyle\sum_{i=1,k} \int_\Gamma \varepsilon_i [\sigma]_{xy,i} \left(\int_{-1}^{+1} < \boldsymbol{W}_a^{2D} + \boldsymbol{\nabla} W_n^{2D} > \right. \\ &\qquad \left. < \boldsymbol{A} + \boldsymbol{\nabla}\phi > dw \right) d\Gamma \end{aligned} \tag{6}$$

where Ω_c^i and ε_i are respectively the domain of the ply i and its thickness, Γ the median surface of the laminate, $\mu_{z,i}$ and $\sigma_{z,i}$ the permeability and the electrical conductivity in the normal direction to the median surface of the material, $[\mu]_{xy,i}$ and $[\sigma]_{xy,i}$ the physical tensors of the material defined in the plan of its median surface. This formulation will be more detailed in the final paper.

IV. APPLICATION TO CFRP COMPOSITES

A 37 plies composite plate is simulated using hexahedron and shell elements. The geometry of these simulations is shown in the Fig. 3 (a). The Fig. 3 (b) gives the comparisons of total induced power calculated in each ply (denoted by its orientation in these figures) for a source current at $5MHz$.

The comparison shows a similar distribution of the induced power obtained with both methods. The stacking sequence is very well taken into account using shell elements. Skin effect is also observed. The distribution of induced power at $5MHz$ is essentially in the upper haft of the laminate.



(a) Circular coil above a plate.

(b) Comparison of the total induced power in each ply.

Fig. 3. A test case.

Taking in to account the eddy-current flow along the thickness of the thin region, degenerated elements can be used to correctly model the behavior of eddy-current in the anisotropic material. Simulation results with CFRP composites give a good accordance with classical method. Moreover, the use of the elements improves significantly the computation time. A time reduction factor of 2 is obtained in the presented simulation.

In the final paper, the degenerated hexahedral elements and their use in $\boldsymbol{A} - \phi$ formulations for eddy-current problem in anisotropic multi-layer regions will be detailed. Comparisons with classical elements will be shown and discussed.

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